

## TABLE OF CONTENTS

<b>SECTION I - MEMORIZATION</b> .....	<b>7</b>
Multiplication Table .....	7
Table of Squares, 1 to 25 .....	8
Table of Cubes - 1 to 12 .....	9
Large Numbers .....	10
Roman Numerals .....	12
Factorials - 0 to 5 .....	13
Order of Operations .....	14
 <b>SECTION II - MULTIPLICATION</b> .....	 <b>15</b>
Introductory Comments to Multiplication Shortcuts .....	15
Multiplication by 5 .....	16
Numbers Ending in 5 .....	17
Multiplication by 11 .....	19
Multiplication by 12, 13, ..., 19 .....	21
Double And Half (Or Larger) Method .....	23
Multiplication by 25 .....	24
Multiplication by 50 .....	26
Multiplication by 75 .....	28
Multiplying Two Numbers That Are Close to 100 But Both Are Less Than 100 .....	29
Multiplication by 101 .....	31
Multiplying Two Numbers That Are Close to 100 But Both Are Greater Than 100 .....	33
Multiplying Two Numbers Close to 100 But One Is Under And The Other Is Over 100 ..	34
Multiplication by 111 .....	35
Multiplication by 125 .....	38
Multiplying Two Numbers That Are Close to 1,000 But Both Are Less Than 1,000 .....	40
Multiplying Two Numbers That Are Close to 1,000 But Both Are Greater Than 1,000 ...	42
Multiplication by 1,001 .....	43
Multiplication by 1,111 .....	45
Multiplication by 3,367 .....	48
Multiplication by 10,101 .....	49
Squaring a Two-digit Number .....	51
The Difference of Two Squares .....	53
The Sum of Two Squares .....	54
Squaring a Three Digit Number .....	56
Multiplication of Two 2-digit Numbers Whose Tens Digits Are The Same And The Units Digits Add up to Ten .....	58
Multiplication of Two 2-digit Numbers Whose Tens Digits Are The Same But Whose Units Digits Are Different .....	59

Multiplication of Two 2-digit Numbers Whose Tens Digits Add up to Ten and the Units Digits Are the Same ..... 60

Multiplication of Two Mixed Numbers Having the Same Rational Parts ..... 61

Multiplication of Two Mixed Numbers Having The Same Whole Numbers And Whose Rational Parts Add up to 1 ..... 62

Multiplication of a Fraction by Its Numerator ..... 63

Subtraction by Way of Multiplication ..... 64

**SECTION III - ESTIMATION ..... 65**

    Introductory Comments on The 5% Rule ..... 65

    Addition & Subtraction Problems ..... 66

    Multiplication Problems ..... 67

    Division Problems ..... 68

    Distributive Property ..... 69

    Multiplication of Three Numbers ..... 70

    Multiplication by Four Numbers ..... 72

    Raising a Number to The 4<sup>th</sup> Power ..... 73

    The Decimal Part of One-seventh ..... 74

**SECTION IV - ROOTS ..... 76**

    Properties of N<sup>th</sup> Root Radicals ..... 76

    Square Roots ..... 77

    Estimating Square Roots ..... 80

    Cube Roots ..... 81

**INDEX ..... 82**

## SECTION I - MEMORIZATION

### MULTIPLICATION TABLE

To use the table find a number in the top row, such as 6, then find a number in the left-hand column, such as 4. The answer of  $6 \times 4$  is found where row and column intersect, in this case 24.

	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>1</b>	1	2	3	4	5	6	7	8	9
<b>2</b>	2	4	6	8	10	12	14	16	18
<b>3</b>	3	6	9	12	15	18	21	24	27
<b>4</b>	4	8	12	16	20	24	28	32	36
<b>5</b>	5	10	15	20	25	30	35	40	45
<b>6</b>	6	12	18	24	30	36	42	48	54
<b>7</b>	7	14	21	28	35	42	49	56	63
<b>8</b>	8	16	24	32	40	48	56	64	72
<b>9</b>	9	18	27	36	45	54	63	72	81

This table tells a lot about the base 10 number system.

- 1) All of the perfect squares (1, 4, 9, 16, ..., 81, 100) lie on the main diagonal from upper left to lower right. Look at the table and notice that  $4 \times 7$  and  $7 \times 4$  equal 28. This illustrates the commutative property of multiplication. Also notice that the products on the right-hand column from top to bottom are the same as those on the last row from left to right.
- 2) All multiples of 5 end in either a 0 or 5.
- 3) All multiples of 10 end in a zero.
- 4) The sum of the digits of all multiples of 3 are a multiple of 3. (i.e.  $3 \times 8 = 24$  and  $2 + 4 = 6$  and 6 is a multiple of 3:  $6 = 2 \times 3$ .)
- 5) The sum of the digits of all multiples of 9 are a multiple of 9(or 3) (i.e.  $9 \times 5 = 45$  and  $4 + 5 = 9$ (or  $9 = 3 \times 3$ .)

Some of these would enable you to quickly check an answer for validity.

## TABLE OF SQUARES, 1 TO 25

After learning your basic multiplication tables, the next step is to memorize the squares of numbers from 1 to 25. Some students memorize the squares of numbers up to 50, but that isn't necessary at this point. Some of the multiplication short cuts will help with the squares of higher numbers. You will know the squares of other numbers larger than 25 because they will be of a particular form or because you will see them quite often. (i.e. there is a short cut for numbers ending in 5 that will be covered, and the squares of 30, 40, 50...up to 250 can be derived from knowing the squares of 3, 4, 5...up to 25 and multiplying times  $10^2$  (100):

$$40^2 = 40 \times 40 = 4(10) \times (4)(10) = 4(4) \times (10)(10) = 16 \times 100 = 1,600.$$

$1^2 = 1$	$11^2 = 121$	$21^2 = 441$
$2^2 = 4$	$12^2 = 144$	$22^2 = 484$
$3^2 = 9$	$13^2 = 169$	$23^2 = 529$
$4^2 = 16$	$14^2 = 196$	$24^2 = 576$
$5^2 = 25$	$15^2 = 225$	$25^2 = 625$
$6^2 = 36$	$16^2 = 256$	
$7^2 = 49$	$17^2 = 289$	
$8^2 = 64$	$18^2 = 324$	
$9^2 = 81$	$19^2 = 361$	
$10^2 = 100$	$20^2 = 400$	

*Note: This table implies that no matter how large a number is, a perfect square cannot have a units digit of 2, 3, 7, or 8.*

Another shortcut is the following:

- 1) Divide the number being multiplied by 125 by 8 and write down the quotient.
- 2) In step 1), you will have a remainder of 0 through 7.
  - a. If the remainder is 0, write down 000 to the right of your quotient in step 1).
  - b. If the remainder is 1, write down 125 to the right of your quotient in step 1).
  - c. If the remainder is 2, write down 250 to the right of your quotient in step 1).
  - d. If the remainder is 3, write down 375 to the right of your quotient in step 1).
  - e. If the remainder is 4, write down 500 to the right of your quotient in step 1).
  - f. If the remainder is 5, write down 625 to the right of your quotient in step 1).
  - g. If the remainder is 6, write down 750 to the right of your quotient in step 1).
  - h. If the remainder is 7, write down 875 to the right of your quotient in step 1).
- 3) This step will be your final answer.

Examples 5 – 8 will do Examples 1 – 4 again but by remainders. I believe doing multiplication by 125 by remainders is faster and easier. It takes practice. You choose the method you like best.

**Example 5. 125 x 42**

- 1) Think,  $42 \div 8 = 5$  with remainder 2.
- 2) Write down 5.
- 3) Remainder 2 implies answer ends in 250 (2/8 of 1,000).
- 4) Write down 250 to the right of 5.
- 5) Therefore,  $125 \times 42 = 5,250$ .

**Example 6. 125 x 61**

- 1) Think,  $61 \div 8 = 7$  with remainder 5.
- 2) Write down 7.
- 3) Remainder 5 implies answer ends in 625 (5/8 of 1,000).
- 4) Write down 625 to the right of 7.
- 5) Therefore,  $125 \times 61 = 7,625$ .

**Example 7. 125 x 147**

- 1) Think,  $147 \div 8 = 18$  with remainder 3.
- 2) Write down 18.
- 3) Remainder 3 implies answer ends in 375 (3/8 of 1,000).
- 4) Write down 375 to the right of 18.
- 5) Therefore,  $125 \times 147 = 18,375$ .

**Example 8. 125 x 768**

- 1) Think,  $768 \div 8 = 96$  with remainder 0.
- 2) Write down 96.
- 3) Remainder 0 implies answer ends in 000 (8/8 = 1.000).
- 4) Write down 000 to the right of 96.
- 5) Therefore,  $125 \times 768 = 96,000$ .