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SECTION I - MEMORIZATION

MULTIPLICATION TABLE

To use the table find a number in the top row, such as 6, then find a number in the left-hand column, such as 4. The answer of 6×4 is found where row and column intersect, in this case 24.

	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

This table tells a lot about the base 10 number system.

- 1) All of the perfect squares (1, 4, 9, 16,..., 81, 100) lie on the main diagonal from upper left to lower right. Look at the table and notice that 4×7 and 7×4 equal 28. This illustrates the commutative property of multiplication. Also notice that the products on the right-hand column from top to bottom are the same as those on the last row from left to right.
- 2) All multiples of 5 end in either a 0 or 5.
- 3) All multiples of 10 end in a zero.
- 4) The sum of the digits of all multiples of 3 are a multiple of 3. (i.e. $3 \times 8 = 24$ and $2 + 4 = 6$ and 6 is a multiple of 3: $6 = 2 \times 3$.)
- 5) The sum of the digits of all multiples of 9 are a multiple of 9(or 3) (i.e. $9 \times 5 = 45$ and $4 + 5 = 9$ (or $9 = 3 \times 3$).)

Some of these would enable you to quickly check an answer for validity.

TABLE OF CUBES - 1 TO 12

After the multiplication tables and squares of numbers to 25, it is useful to memorize the cubes of numbers from 1 to 12.

$$1^3 = 1$$

$$7^3 = 343$$

$$2^3 = 8$$

$$8^3 = 512$$

$$3^3 = 27$$

$$9^3 = 729$$

$$4^3 = 64$$

$$10^3 = 1,000$$

$$5^3 = 125$$

$$11^3 = 1,331$$

$$6^3 = 216$$

$$12^3 = 1,728$$

Note:

- 1) *The numbers 1, 64, and 729 are also perfect squares.*
- 2) *The number 1,728 represents the number of cubic inches in a cubic foot, and the number 27 represents the number of cubic feet in a cubic yard.*
- 3) *The number 231 is not a perfect cube, $231 = 11 \times 21 = 11 \times 7 \times 3$, but the number represents the number of cubic inches in one gallon.*
- 4) *The units digit of a perfect cube can be any number.*

MULTIPLYING TWO NUMBERS THAT ARE CLOSE TO 100 BUT BOTH ARE LESS THAN 100

In this short cut, we are assuming that both whole numbers are greater than 90 or the product of their differences from 100 is less than or equal to 99.

The short cut is as follows:

- 1) Subtract each number from 100.
- 2) Multiply the two numbers in step 1) together and write down their product. If the product is not a 2-digit number, place a zero to the left of the product making it a 2-digit number.
- 3) Take either one of the differences in step 1) and subtract it from the other number.
- 4) Place this difference from step 3) to the left of your answer in step 2) and you will have your answer.

Example 1. 93 x 98

- 1) $100 - 93 = 7$ and $100 - 98 = 2$.
- 2) $7 \times 2 = 14$.
- 3) Write down 14.
- 4) $98 - 7 = 91$ or $93 - 2 = 91$ (Just one difference; they are always the same number).
- 5) Write down 91 to the left of 14.
- 6) Therefore, $93 \times 98 = 9,114$.

Example 2. 89 x 94

- 1) $100 - 89 = 11$ and $100 - 94 = 6$.
- 2) $11 \times 6 = 66$.
- 3) Write down 66.
- 4) $94 - 11 = 83$ or $89 - 6 = 83$.
- 5) Write down 83 to the left of 66.
- 6) Therefore, $89 \times 94 = 8,366$.

Example 3. 97 x 98

- 1) $100 - 97 = 3$ and $100 - 98 = 2$.
- 2) $3 \times 2 = 6$.
- 3) Write down 06 (a 2-digit product).
- 4) $98 - 3 = 95$ or $97 - 2 = 95$.
- 5) Write down 95 to the left of 06.
- 6) Therefore, $97 \times 98 = 9,506$.

SQUARE ROOTS

The principal (positive) square root of a positive real number x is denoted by \sqrt{x} ; the root index is 2 but it is not generally written as $\sqrt[2]{x}$. For square roots (or any even root index), the radicand must be greater than or equal to zero, in order to have a real number answer. That is, $\sqrt{4} = 2$ but $\sqrt{-4}$ is not a real number. Right now, all numbers and answers to a problem are real numbers. Later, in the section on complex numbers, we will discuss the square root of a negative number.

If we want the answer to the square root of a positive real number x ,

- 1) to be negative, we will write it as $-\sqrt{x}$ or
- 2) to be positive or negative, we will write it as $\pm\sqrt{x}$.

If x is any real number then $\sqrt{x^2} = |x|$ where $|x|$ denotes the absolute value of x . By this we mean, $\sqrt{(-3)^2} = |-3| = 3$.

On a number sense test or to work with square roots in general, you should know your squares through 25.

From the section, Properties of Nth Root Radicals, we can write the following.

- 1) $\sqrt{13^2} = 13$ and $\sqrt{(-81)^2} = 81$.
- 2) $\sqrt{16} = (16)^{1/2} = 4$ and $\sqrt{5} = 5^{1/2}$.
- 3) $\sqrt{3^4} = 3^{4/2} = 3^2 = 9$.
- 4) $9^{3/2} = (\sqrt{9})^3 = (3)^3 = 27$.
- 5) $\sqrt{4^5} = 4^{5/2} = (2^2)^{5/2} = 2^5 = 32$ or $(\sqrt{4})^5 = 2^5 = 32$.
- 6) $\sqrt{44} = \sqrt{4} \sqrt{11} = 2\sqrt{11}$ (Product Rule).
- 7) $\sqrt{72} = \sqrt{36} \sqrt{2} = 6\sqrt{2}$.
- 8) $\sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3}$ (Quotient Rule).
- 9) $\sqrt{\frac{27}{16}} = \frac{\sqrt{27}}{\sqrt{16}} = \frac{\sqrt{9}\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$.